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Fuzzy concepts in production management research: a review

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The use of fuzzy methodologies is an efficient way of accounting for vagueness in human judgment. This paper illustrates potential applications of fuzzy methodologies to various areas of production management including new product development, facilities planning, human production management, production scheduling and inventory control, quality and industrial process control, and cost/benefit analysis. In addition, important areas for new research are discussed.

Introduction

Production systems are defined by Starr (1978) as 'working arrangements of people, material, energy, and machines, whereby agreed upon forms of work are accomplished'. 'Good' management of such systems implies the efficient coordination of the above entities so that the system remains productive. One important aspect of management is the decision making function. For example, decisions concerning both the allocation of scarce resources (i.e. people, material, energy and machines) to a place and time, and the determination of policies and procedures must be made.

From an operation research/management science perspective, decision making is facilitated through the use of an explicit, and usually computerized, model (i.e. a crisp model). For example, there have been numerous models and corresponding solution algorithms developed for the production scheduling function (Graves 1981). Such models can be classified as being either descriptive or prescriptive in nature. Descriptive models can be used to forecast the effect of a decision on a measure (e.g. cost), or measures, of interest. Prescriptive models, when solved, will output an 'optimal' decision (e.g. that production schedule that minimizes cost) based on one or more objectives. Hence, these mathematical models can be characterized by:

(*a*) decision variables

(*b*) objective function(s) of the decision variables and

(*c*) constraint functions of the decision variables which are restricted in some fashion.

Models are formulated through two sources:

(*a*) the perceptions of the decision maker(s) (i.e. managers) about the problem situation

(*b*) data gathered about the problem situation.

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The first source might be termed the decision maker's *mental model* of the situation. These sources are interdependent—in particular they are interactive over time. For example, the decision maker's perceptions affect the types and amount of data which he decides to gather over time, and the results of data collection can affect the decision maker's perceptions about the problem (see Fig. 1).

There are several difficulties, at least from the decision maker's perspective, with such a procedure for the development of a mathematical model. In particular, the decision maker's mental model is characterized by vagueness and imprecision in several aspects. This vagueness, which is a result of the complexity of typical production systems, can be characterized by questions such as:

(*a*) What should the objective function (or objective functions) be for the model?

(*b*) Given more than one (conflicting) objective, which can be measured through the use of a multidimensional outcome vector space, what is the preference order over this vector space?

(*c*) How should the constraint functions be restricted?

(*d*) What are the functional relationships between the decision variables and the objective and constraint functions?

To some extent, vagueness related to answering the last question can be reduced through the data collection phase; however, vagueness still remains. As examples of the expression of vagueness, consider the following statements:

1. I *think* that a system design which results in a cost of $10 000 and a reliability of 0·9 would be preferable to a design which results in a cost of $11 000 and a reliability of 0·92 (question *b*).
2. There should not be any *large* differences in production output from one month to the next over the next year (question *c*).

In effect, the crisp mathematical models are not capable of incorporating such vagueness—hence, the decision maker feels uncomfortable with their use.

The theory of fuzzy sets (Zadeh 1965) admits the existence of a type of uncertainty due to vagueness (i.e. fuzziness) rather than due to randomness alone, where classes of objects (labels of fuzzy sets) have gradual rather than abrupt transition from membership to non-membership. The above discussion suggests several reasons for the use of fuzzy methodologies in production management research. First, in many cases the experience and managerial judgment inherent in a mental model must supplement the use of structured knowledge provided by

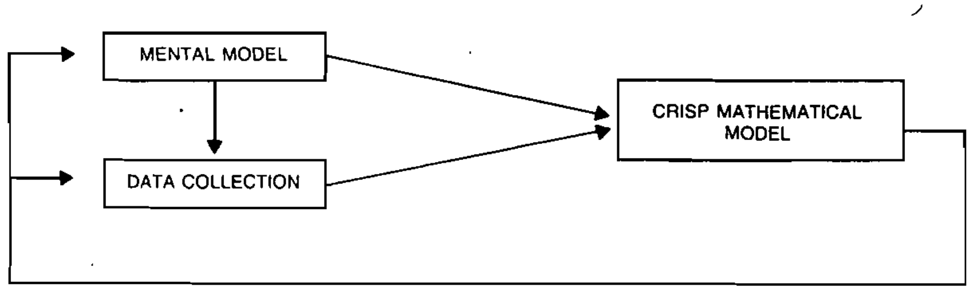

Figure 1. Interdependence of mental model, data collection and mathematical model.

decision theories. Second, most of the decisions to be made in production management systems are decisions made in a fuzzy environment (Zimmerman 1976, Tanaka *et al.* 1977) where the objective functions, constraints, and decision variables are not completely defined and cannot be precisely measured. Third, our lack of understanding of the phenomena under investigation limits the application of conventional models. Finally, in many instances the quality of available information is degraded by inexactness and vagueness due to personal bias and subjective opinion and the need for evaluation of non-distributive uncertainties. As indicated by Negoita (1981) 'the ability to work with fuzzy tolerances, instead of numbers, seems to be exactly the way people cope with their environment'.

In the remainder of this paper, we will review how fuzzy methodologies (e.g. fuzzy relations, linguistic variables, fuzzy numbers, and fuzzy preferences) can be employed in the decision making process for various areas of production management, including:

(*a*) new product development
(*b*) facilities planning
(*c*) human production management
(*d*) production scheduling and inventory control
(*e*) quality and industrial processes control
(*f*) cost/benefit analysis.

In addition, we briefly discuss the use of fuzzy methodologies in mathematical programming, an important tool for production management. Prior to the review, we introduce some basic concepts in fuzzy methodologies in the next section of the paper. Finally, in the last section of the paper, we present conclusions and needs for further research.

Basic concepts in fuzzy methodologies

Definitions of basic concepts in fuzzy methodologies are presented by Zadeh (1973).

A fuzzy subset A of a universe of discourse U is defined by a membership function $f_A : U \rightarrow [0, 1]$ which associates with each element u of U a number $f_A(u)$ in the interval $[0, 1]$, where $f_A(u)$ represents the grade of membership of u in A. Formally, A can be written as:

$$A = \{(u, f_A(u)),\ u \in U\} \tag{1}$$

The support of A is the set of points in U at which $f_A(u)$ is positive. An α-level set of A is a non-fuzzy set denoted by A_α which contains all elements of U whose grade of membership in A is greater than or equal to α. A is called normal if there is x such that $f_A(x) = 1$.

To simplify the notation, a fuzzy subset A with discrete membership function can be expressed as follows:

$$A = f_1/u_1 + f_2/u_2 + f_3/u_3 + \cdots + f_n/u_n \tag{2}$$

where f_i, $i = 1, 2, 3, \ldots n$, is the grade of membership of u in A, and

$$U = u_1 + u_2 + u_3 \cdots + u_n$$

U is the universe of discourse, where $+$ denotes *union* rather than the arithmetic sum.

For the given fuzzy subsets A and B one can perform basic operations as follows:

Complement: $\bar{A} \Leftrightarrow f_A = 1 - f_A$

Union of two fuzzy subsets A and B, denoted $A \cup B$:

$$A \cup B \Leftrightarrow f_{A \cup B} = f_A \vee f_B$$

Intersection of two fuzzy subsets, denoted $A \cap B$:

$$A \cap B \Leftrightarrow f_{A \cap B} = f_A \wedge f_B$$

where $\vee$ and $\wedge$ denote MAX and MIN operators, respectively.

Product of A and B: $A \times B \Leftrightarrow f_{A \times B} = f_A f_B$

Concentration of A: $\text{CON}(A) \Leftrightarrow A^2 \Rightarrow \text{CON}(f_A) \Leftrightarrow f_A^2$

Dilation of A: $\text{DIL}(A) \Leftrightarrow A^{0\cdot 5} \Rightarrow \text{DIL}(f_A) \Leftrightarrow f_A^{0\cdot 5}$

Contrast intensification of A, denoted by INT (A):

$$\text{INT}(f_A) \Leftrightarrow \begin{cases} 2 f_A^2 & \text{for } 0 \leqslant f_A \leqslant 0\cdot 5 \\ 1 - 2(1 - f_A)^2 & \text{for } 0\cdot 5 \leqslant f_A \leqslant 1\cdot 0 \end{cases}$$

is the operation that increases these values of f_A which are above 0·5 and decreases those which are below that point.

The concept of fuzzy events (Zadeh 1968, Okuda *et al.* 1978)

Let X be a set of events $(x_1, x_2, \ldots, x_n)$ with probabilities $p(x_i)$. A fuzzy event A is a fuzzy set on X which membership function f_A is measurable and in a discrete case:

$$P(A) = \sum_{i=1}^{n} f_A(x_i) p(x_i) \tag{3}$$

where $P(A)$ is called the probability of fuzzy event A. Such defined $P(A)$ can be interpreted as the expectation of the membership function of a fuzzy event.

The amount of uncertainty associated with fuzzy event can be represented as the entropy measure $H(A)$ of a fuzzy subset A of the finite set $(x_1, x_2, \ldots, x_n)$ with respect to a probability distribution $P = (p_1, p_2, \ldots, p_n)$ as follows:

$$H^P(A) = - \sum_{i=1}^{n} f_A(x_i)\, p_i \log p_i \tag{4}$$

This entropy measure could be used in system analysis as an assessment of the value of information describing given condition(s).

The linguistic approach (Zadeh 1973, 1975)

The linguistic characterization uses a concept of a linguistic variable with values which are not numbers but words (or sentences) of a natural (artificial) language. A linguistic value (e.g. 'high') is interpreted as a label for a fuzzy restriction on the values of the base variable. The possible labels, like: 'high', 'moderate' or 'low' are regarded here as the linguistic values of the variable. The linguistic values play the same role as the numerical values, but are much less precise.

The fuzzy restrictions on the values of the base variable are characterized by the compatibility functions. Each such function associates with each value of the base variable a number in the interval [0, 1] representing the compatibility with the fuzzy restriction.

The compatibility function (even if it is not objective) can be standardized for particular values of a given linguistic variable. For example, the compatibilities of the numerical cost of $1 m with the fuzzy restrictions labeled 'low' and 'high' could be 0·1 and 0·85, respectively.

Typical values of the linguistic variables contain not only the primary terms (like 'low' or 'average') but also hedges such as 'very' or 'more or less'; fuzzy connectives: 'and', 'or' and negation 'not'. The hedges, connectives and negation are used as modifiers of the operands (primary terms) in a context-dependent situation. The modification of the meaning of primary terms (e.g. for $f(u)$) can be done as follows:

$$\text{very low} = \text{low}^2 \Leftrightarrow f^2(u)$$

$$\text{not low} = 1 - \text{low} \Leftrightarrow 1 - f(u)$$

$$\text{more or less high} = \text{high}^{0 \cdot 5} \Leftrightarrow f^{0 \cdot 5}(u)$$

$$\text{extremely high} = \text{very (very high)} = \text{high}^4 \Leftrightarrow f^4(u)$$

Fuzzy implication and compositional rule of inference (Zadeh 1975)

The approximate reasoning utilizes fuzzy conditional statements and compositional rules of inference. In a fuzzy conditional statement: If A then B, or $A \Rightarrow B$, antecedent (A) and consequent (B) are fuzzy sets rather than propositional variables. Statement of this kind describe a fuzzy relation R between two fuzzy variables A and B.

If A is a fuzzy subset in a universe of discourse U, and B is a fuzzy subset in a universe of discourse V, then the cartesian product of A and B, $A \times B$, is defined as a fuzzy relation R from U to V:

$$A \times B = \sum_{R = U \times V} (f_A(u) \wedge f_B(v))/(u, v)) \tag{5}$$

where R is usually given in the form of a matrix.

A fuzzy conditional statement can be represented in terms of the cartesian product as follows:

$$A \Rightarrow B = \text{IF } A \text{ THEN } B \Leftrightarrow (A \times B + \bar{A} \times V) \tag{5}$$

where + denotes union of fuzzy relations.

In case of a conditional statement of IF A THEN B ELSE C we will have the following:

$$\text{IF } A \text{ THEN } B \text{ ELSE } C \Leftrightarrow (A \times B + \bar{A} \times C) \tag{7}$$

If the fuzzy relation R from U to V is known, and A is a fuzzy subset of U, then the fuzzy subset B of V is induced by A, is given by the composition of R and A

$$B = A \circ R \tag{8}$$

where B is given by the max–min product of A and R:

$$B = A \circ R \Leftrightarrow f_B(v) = \sup_{v} (f_A(u) \wedge f_R(u, v))/(v) \tag{9}$$

Fuzzy numbers (Dubois and Prade 1980)

A fuzzy number is a fuzzy subset of the real line defined by its membership function $f: R \rightarrow [0, 1]$. Any normal fuzzy number can be represented as a 4—tuple (a, b, α, β) where α- is called the 'left bandwidth', and β the 'right bandwidth', and $[a, b]$ is the closed interval on which the membership function is equal to 1·0. The shape of the left and right slopes of the membership function can be limited to an even function S, such that $S(-x) = S(x)$, and $S(O) = 1$. Several operations such as addition, subtraction, multiplication, division and exponentiation, allow one to perform calculations on fuzzy numbers similar to those performed on non-fuzzy numbers in crisp algebra.

Extension principle (Zadeh 1975)

An extension principle allows any non-fuzzy function to be 'fuzzified'. If the function's arguments are represented as fuzzy sets, then the value of such function is also a fuzzy set with a unique membership function.

Let the arguments of the function G be $X_1, X_2, \ldots, X_n$ (denoted as $\bar{X}$), and let the membership functions associated with $\bar{X}$ be given as $f_1(x_1), f_2(x_2), \ldots, f_n(x_n)$. Note that these membership functions allow the representation of the fuzziness associated with the realization of $X_1, X_2, \ldots, X_n$. If G is a mapping from $X_1, \times X_2, \ldots, \times X_n$ to Y, then the fuzzy image of G is given as

$$f_{G(\bar{X})}(y) = \sup_{\bar{X}} \left\{ \min_{i=1,n} f_i(x_i) \right\} \tag{10}$$

such that $G(\bar{X}) = y$.

The extension principle is one of the most important tools of fuzzy sets theory since it provides for a transition between non-fuzzy elements and fuzzy concepts.

Theory of possibility (Zadeh 1978)

If B is a fuzzy subset of a universe of discourse $U = \{u\}$, and B is characterized by the membership function f_B, then the proposition of the form 'X is B', where X is a variable taking values in U, induces a possibility distribution Π_x which equates the possibility of X taking the value u to $f_B(u)$ i.e. the compatibility of u with B is denoted by

$$\Pi(X = u) = \Pi_x(u) \Leftrightarrow f_B(u) \tag{11}$$

Set B can be treated as a fuzzy restriction on X since it acts as a constraint on the values that may be assigned to X.

If $R(X)$ denotes a fuzzy restriction associated with X, then one can write the above in the form of a relational assignment equation, as follows:

$$R(X) = B \tag{12}$$

The possibility distribution Π_x satisfies the equation $\Pi_x = R(X)$ and the possibility that $X = u$, $\Pi_x(u)$, is equal to $f_B(u)$. Thus, X becomes a fuzzy variable which is associated with the possibility distribution Π_x in the similar way as a random variable is associated with a probability distribution. It should be remembered that according to possibility/probability consistency principle‡ a variable may be associated both with a possibility distribution and a probability distribution.

Applications

The abstract concepts discussed above can prove useful in many aspects of production management research. These aspects are discussed in detail in the following sections.

New product development

At certain stages of a new product's development, management must decide whether or not to continue the introduction process. The basis for this decision is usually insufficient information about the complex and uncertain environment (Nojiri 1982). As observed by Conrath (1973) the state of information is less than complete, and the probability distributions (applied when using statistical models for uncertainty) are based upon only the 'best guesses', resulting in fuzzy results. Moreover, the decision makers often prefer simple choice models rather than the complex ones to support their judgements.

To deal quantitatively with imprecision of the managerial judgment stated in a natural language, the concept of fuzziness was proposed (Nojiri 1982). Fuzzy sets of type II (where grades of membership are also distinct fuzzy sets) and a team theory was used to formulate the decision problem. The decisions were based on managerial evaluations of the degree to which the product and supporting strategy could be expected to be successful and the degree of confidence the management had in those evaluations. A computer-based system allowing dialogue in a natural language regarding the new product introduction decisions was proposed.

Facilities planning

Facilities planning, as an important function of production management, is a composite of facilities location and facilities design, including layout design and material handling system design (Tompkins and White 1984). As such, facilities planning is aimed at the effective utilization of people, equipment and space. The location of a facility refers to its placement with respect to existing (or new) other

‡ The possibility/probability consistency principle (Zadeh 1978) states that 'probable implies possible' (what is probable must be possible, but not vice versa).

facilities (production units). Material handling involves handling, storing and controlling of material flow in terms of quantity, position, place, movement and time.

Many different quantitative models have been developed to aid the decision makers with facilities planning tasks. Such models are either deterministic or probabilistic. These refer to location models, warehouse layout models, conveyor models, and storage and waiting models. Location models are based on the measured distances between existing and new facilities, expressed as rectilinear or euclidean measures. Location–allocation models are usually solved by employing enumeration procedures. Linear or quadratic assignment models attempt to determine the optimal location of multiple new facilities in a discrete space. Other problems under consideration lead to development of models for determination of the location of products for storage in a warehouse.

Many variables or relationships relevant to the above models are initially specified in an imprecise and vague manner and later these are simplified for ease of analysis in an attempt to eliminate or reduce fuzziness. For example, the distances between planned facilities may be expressed as being 'short', 'medium' or 'long'. In some instances, it may be even beneficial in a design process to develop and utilize such verbal descriptors of the distance magnitude, rather than using strict values as approximations for the desired magnitudes.

Another example of a planning problem in which imprecision and subjective evaluations play an important role is the procedure for systematic facility site selection. The procedure includes both quantifiable and non-quantifiable, and objective and subjective factors, as well as evaluations of these factors' measures and weights. The evaluation of the critical factor measures (the ones for which absence or presence precludes the location of a plant at a site regardless of other conditions) is based upon the critical factor index (CFI).

At present, the CFI can be assigned either a value of 0 or 1, depending on whether or not the site meets the minimum requirement with respect to a given critical factor. In practice, however, experts would be more inclined to use verbal descriptors since the criteria for meeting specific requirements are not sharply defined. Therefore, the availability for labor could be best described as 'low' instead of 0. At the same time, availability of utilities or transportation could be 'high' or 'medium' but not 1. Furthermore, under traditional methods of evaluation, the assignment of a dichotomous value of 0 or 1 for the 'medium' condition would be unreasonable.

The determination of subjective factor weights, as measures of the relative importance of the factor in the location analysis, also introduces imprecision due to fuzziness. However, the use of conventional preference theory excludes the possibility of the application of such soft preference statements as: A is 'strongly' or 'much less' preferred than B. Since the degree of preference is imprecise due to nondistributional subjectiveness, it should be treated as a fuzzy category.

An interesting example of the unconscious use of fuzzy concepts in facilities planning can be found in material handling problems. The principles of material handling are usually expressed as 'rules of thumb' which are formulated by experienced managers (Tomkins and White 1984). The fuzziness of such rules is due to use of vague statements like 'excessive material on hand', 'long hauls', 'crowded deckspace', 'high' indirect payroll or 'hazardous lifting by hand'. The meaning of such adjectives as 'excessive', 'long' or 'high' is very familiar to those who are constantly using these terms, and can be effectively modelled using the linguistic approach. A fuzzy expert system utilizing fuzzy descriptors and fuzzy logic could be developed to provide the knowledge-based expertise in the area of material handling and planning.

Human-production management systems

Management and human factors

The main function of an industrial engineer in a production environment is that of management (Saunders 1982). A special emphasis must be placed on the human-related components of the management process. The need to account for natural imprecision and vagueness of the human-production system has forced researchers to look for new modeling techniques for such areas of interest as selection and training of human resources, work systems design and performance measurement.

Fuzziness in human-production management systems stems from:

(*a*) the manager's inability to acquire and process adequate amounts of information about the behaviour of a particular subsystem (subjective fuzziness)

(*b*) the vagueness of rules governing the complex, human-production management system itself (objective fuzziness).

Regarding the subjective category of fuzziness, it should be noted that our knowledge about the behavior of the human element of the system is still incomplete and imprecise—for example, the knowledge about the perception of human effort or the relationship between the operator's sensations and his physical and mental stimuli. Therefore, the task of quantifying human stress responses due to physical, behavioral or psycho-social factors for the purpose of using them as parameters or variables in the decision-making process, is a very difficult one.

Another important issue in human-production management systems is lack of norms for mental tasks. Although the work performance standards exist for most of the 'physical' production tasks, there are no such universal standards for the tasks that will dominate the market in the future and which are predominantly mental in their character. A linguistic approach was recently used (Mital and Karwowski 1984) for the development of performance standards which takes into account both mental and physical components of work.

From the production management point of view, development of such performance measures and their use in everyday practice is a necessary phase of production control and related decision-making processes. In this way the second category of fuzziness (i.e. objective fuzziness) can be explicitly taken into account, rather than just being implied during the process.

Management of personnel

Effective management of human-production systems requires accounting for the effect of present and future workforce on this process. Several mathematical models have been proposed to optimize decisions related to recruitment, selection and promotion of personnel. As indicated by Ollero and Freire (1981) 'personnel management criteria are imprecise due to the complex nature of the requirements and the difficulties to deal with personnel characteristics'.

Testing and placement programmes, personnel selection, training and promotion policies, as well as performance appraisal are of great importance to production managers since sound personnel management is a prerequisite for effective performance of any production unit.

The use of subjective measures (continuous or discontinuous scales) for the evaluation of an employee's characteristics or checklists comparing types of desired behaviour patterns and job performance, is a common practice in personnel management. Such methods are aimed at quantification of qualitative information and therefore must incorporate imprecision and vagueness. Fuzzy set theory would seem to be an efficient tool for considering such imprecision and vagueness.

Ollero and Freire (1981) used fuzzy representations of the relations between people and required characteristics when studying the recruitment and promotion of personnel. A fuzzy approach to this problem was characterized by four basic steps:

(*a*) definition of an adequate referential set of fuzzy characteristics

(*b*) definition of a membership function of the required profile P_0

(*c*) finding the membership functions of the candidates' profiles $P_1, P_2 \ldots, P_n$

(*d*) computing the selection coefficients to obtain a final classification.

The membership functions were derived from standard tests and expert evaluation. When comparing job requirements (profile P_o) and the candidates' profiles, some of the exact values had to be fuzzified according to the extension principle.

Van Velthoven (1977) proposed a fuzzy promotion model that satisfies the requirement of finding an objective decision rule, allowing x of n ($x < n$) individuals to be promoted based on the general requirements of the higher profile and the efficiency in the achievement of the actual functions. The binary fuzzy relations were defined as based on the concordancy conditions. It was concluded that for promotion problems with disjoint criteria the fuzzy operator of intersection was the most suitable one.

Production planning and inventory control

The process of simultaneously analysing work force size, production rate, and inventory control policies is known as aggregate planning and scheduling. In this section of the paper, we concentrate on production planning and inventory control. The performance of each of these functions and the critical parameters of each are dependent upon the type of production system under consideration: the flow shop, the job shop, or the project-oriented system. In addition, an important parameter in the production system is the demand over time for the material under consideration. This demand can be viewed as being either dependent upon or independent of the demand for other items.

A material requirements planning (MRP) system is often employed to determine production planning and inventory control policies for dependent demand items (e.g. items that are components for other items). This MRP approach is typically employed in job-shop systems, and less often in project-oriented systems. The idea is to obtain the gross demand over time for the end items of which this item is a component. This gross demand over time is considered along with stock on hand in order to generate the net requirements over time. These net requirements are considered along with lead times and various costs (e.g. set-up costs) in order to determine the sizes of the lots of items to be released and the timing of those releases over time. (See Fig. 2 for a schematic of this process.)

Vagueness is inherent in several parameters of this problem—in particular in the stock on hand, the set up and variable production costs and the lead times. Each of these quantities could be modelled as fuzzy numbers in order to arrive at membership functions associated with desirable lot sizes and release times. Of particular interest here is the uncertainty associated with the lead times. As noted by Negoita (1981), lead times can be decomposed into paperwork time, set-up time, processing time, move time, and queue time; but queue time typically makes up at least 90% of the total processing time. This queue time, which is a function of shop-load from other production requirements, is usually extremely difficult to estimate in an accurate fashion. Hence, the uncertainty inherent in the overall lead time is not of the probabilistic type.

Very few, if any, examples exist of the application of fuzzy methodologies to production planning and inventory control for dependent-demand items. One reason for this is that the problem is typically viewed from a data processing orientation (in which large amounts of data are processed and simple calculations are performed) as opposed to a mathematical modelling orientation. Hence, the techniques of operations research in general are not thought of as being particularly useful in this area (at least in practice). However, the vagueness associated with the parameters of the problem, in particular the lead times, would indicate that the use of fuzzy methodologies in this area may very well be beneficial.

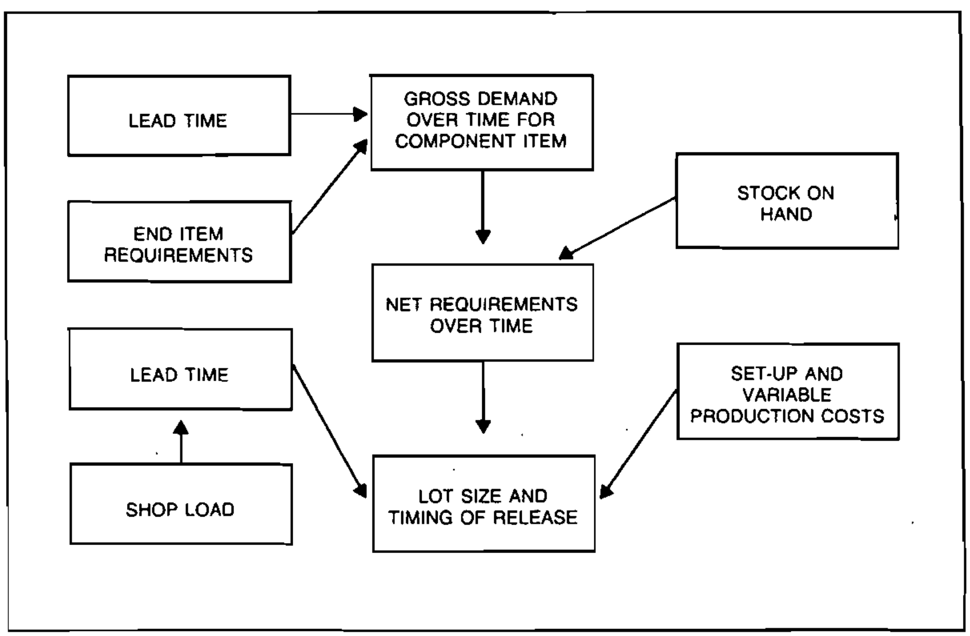

Figure 2. Production planning and inventory control for dependent demand items.

The techniques of operations research have been applied more frequently when the orientation is towards 'independent-demand items' (e.g. items purchased by customers). For example, consider the problem of inventory control for an independent-demand item. A typical objective of a mathematical model is the minimization of the cost associated with the inventory policy. The three major categories of cost are the ordering cost, the carrying cost, and the cost associated with being out of stock. Other important parameters of this problem include the projected demand over time for the item and the lead time associated with the item. These parameters are extremely difficult to estimate accurately, especially the carrying cost, the cost of being out of stock, and the projected demand. Hence, the use of fuzzy methodologies may prove useful here.

Both Sommer (1981) and Kacprzyk and Staniewski (1982) address the problem of incorporating fuzziness into the modelling and analysis of production planning and inventory control problems. Sommer applies fuzzy dynamic programming (Bellman and Zadeh 1970) to an inventory and production scheduling problem in which the management wishes to fulfill a contract for providing a product, and then withdraw from the market. Hence the objectives of the problem can be represented as fuzzy statements:

(*a*) the production should decrease as continuously as possible

(*b*) the stock should be at best zero at the end of the planning horizon.

The planning horizon is separated into discrete planning periods 1, 2, ..., T and the problem becomes one of determining the production levels and hence the stock on hand, for each of the planning periods, given as input the amount of the product that must be provided during each period. The fuzzy objectives, as expressed above, can in turn be expressed as a membership function of the production levels for each period. This membership function represents the set of desirable production levels to fulfill the objectives stated above. Sommer (1981) notes that an optimal set of production levels is that set which maximizes this membership function.

Kacprzyk and Staniewski (1982) consider the fuzziness inherent in the inventory level, the demand, the replenishment, the constraints imposed on replenishments and the goals for desirable inventory levels in an infinite horizon problem. The output of the algorithm developed for solving this fuzzy model is the firm's optimal time-invariant replenishment rule as represented by a fuzzy conditional statement (conditioned on current inventory levels). The algorithm employs a procedure used in fuzzy mathematical programming in which a fuzzy optimal decision is viewed as the intersection of fuzzy constraints and a fuzzy goal. In addition, the computational burden imposed by the use of the algorithm is lessened through the use of approximation by reference fuzzy sets (Wenstop 1976).

Both of these approaches to fuzzy production scheduling and inventory modelling employ what might be termed a high level approach to the modelling of imprecision. That is, instead of modelling the vagueness associated with carrying costs, out-of-stock costs etc. the vagueness associated with desirable inventory levels is modelled. Hence the imprecision associated with these various inventory costs are modelled only implicitly. Kacprzyk and Staniewski (1982) note a reason for this procedure in their paraphrasing of Hadley and Whitin (1963): 'a transformation of a cost-optimization problem into one of maintaining some desired inventory level is consistent with inventory theory'. It seems though that a more detailed analysis of the fuzziness inherent in these problems would warrant further investigation.

With respect to the scheduling of individual activities in projects, Chanas and Kamburowski (1981) advance the use of fuzzy PERT (FPERT). In this procedure the individual activity durations are represented as fuzzy numbers because of their inherent imprecision. Therefore the algorithm requires the addition of fuzzy numbers in order to arrive at 'fuzzy variable possibility distributions' for the various event times in the network.

Prade (1980) suggests the use of fuzzy concepts in queueing analysis in an exploratory paper. He suggests that often it is useful to view some parameters (e.g. mean service time) in a queueing model as being probabilistic and others as being possibilistic in nature. The uncertainty associated with the probabilistic parameters is a result of randomness, while the uncertainty associated with the possibilistic parameters is a result of vagueness. Prade suggests a procedure for considering both the possibilistic and the probabilistic aspects of the problem in the analysis. The outputs from such an analysis are fuzzy probabilities for various performance parameters.

Finally, it can be noted that managers are apprehensive about using the existing mathematical models for aggregate production planning, and instead rely more on the use of their own heuristics or intuitive decision rules. These judgmental models work very well in practice. As indicated by Rinks (1981), fuzzy conditional statements can be effectively used to model the relationships among sales forecasts, work force levels, inventory levels, changes in work force and production levels. Such statements can then be combined using the compositional rules of inference to form a model for aggregate production planning.

Two fuzzy algorithms have been developed for production and work force planning (Rinks 1981). Both of these consist of a series of relational assignment statements (linguistic rules). The values of linguistic variables were derived from exponential functions. When making the decisions concerning production levels and changes in the work force, human judgment and experience were used to specify the cost structure. The derived fuzzy algorithms for aggregate production planning were then validated. The results of fuzzy algorithm simulations have shown that for situations where sufficient data is not available to estimate cost functions with reasonable precision, a manager's decision to use the judgmental model (rather than a mathematical model) can be fully justified.

Quality and industrial process control

A fuzzy interpretation of economic control limits

Acceptance control charts with control limits that are based on the specification limits for a product rather than the average of a controlled process, are widely used in practice. Application of these acceptance control charts to a specific product characteristic imposes a dichotomous categorization of conformance with the quality standard, where the product is either good or bad, regardless of how defective it is. In other words, it is usually assumed that the conformance criterion is a crisp category with no room for graded conformance. Such an approach seems to be very rigid since it does not allow one to account for the severity of product nonconformance. Consequently, its use may prove to be costly when the material costs are high and much labour has already been invested in the product.

Bradshaw (1983) has proposed a quantitative representation of graded nonconformance based on the assumption 'that costs resulting from substandard quality are sometimes related to the extent of nonconformance'. If the relationship between a particular quality measure and incurred costs is known, the grade of nonconformance associated with the quality characteristic can be expressed in terms of a fuzzy compatibility function. The degree of nonconformance is scaled over the [0, 1] interval, and the acceptance control chart with fuzzy economic control limits is built. The membership function for a fuzzy set of nonconformance may be specifed by relating the prohibitive cost (the cost where rework costs equals the scrap cost) with the cost of scrapping the part.

The fuzzy control limits are described using fuzzy bands equal to the difference between the value of the quality characteristic where the nonconformance cost is maximum and the value where it is zero (the specification limit of an ordinary control chart). It is the task of the manager, however, to determine the width of the fuzzy nonconformance band(s). Bradshaw (1983), who apparently did not make a single reference to the fuzzy literature, suggested that the appropriate width could be found by observing the graph of the curve for the quality characteristic being charted.

Even if such an approach does fuzzify the control limits, it still leaves a question of determining the appropriate width of the bands, which itself is a fuzzy concept. Therefore, it seems more general to express the ordinary control limits as fuzzy numbers. In this instance the width of the bands would be intrinsically taken into account when building a control chart. Also, it is unlikely that the relationship between the grade of (non)conformance and the cost of reworking the part would be linear as suggested. The use of a linguistic approach for the quantification of quality characteristics (in terms of degree of conformance) as well as associated economic costs could permit a more natural communication between lower management at the production level and the quality control unit.

Fuzzy control of production processes

Control of many important production processes constitutes a continuous challenge for the production management. Since these processes are usually very complex, modern control theory has failed to provide useful mathematical models for their representation (Tong 1977). In particular, the difficulties associated with building successful models are due to the following:

(*a*) the state variables or the control variables cannot be expressed quantitatively
(*b*) the identification of the production process-system structure (or parameters) is impossible or very expensive
(*c*) the control conditions are very complex, and their importance with respect to the objective of control are not clearly stated (Pun 1977).

Moreover, the reliability of such control processes cannot be satisfactorily evaluated due to usually imprecise and often incomplete information about the data input.

Nevertheless, many such complex processes are being successfully controlled by experienced process operators, who gained their knowledge during years of practice on the job. While this is fortunate, one cannot simply rely on the premise that there will always be man-power available to fulfill the requirements of the complex production control tasks. Therefore, attempts have been made to utilize the knowledge of human operators, and to represent their qualitative models of a system's performance in the form of production rules.

In order to account for such imprecision of human thought the linguistic approach (Zadeh 1973, 1975) was applied. The idea of a fuzzy logic controller was also successfully introduced: (Mamdani and Assilian 1975)—control of a steam engine; (Kickert and Lemke van Nauta 1976)—control algorithm for a warm water plant; (Tong 1977)—control of a pressurized tank containing liquid. All of these models are rule-based fuzzy controllers, utilizing the situation-action pairs in the form of: IF (condition) THEN (action) ELSE (...) statements and their logic connectives OR, AND, NOT, etc.. The rule-based decision making processes implicitly accept the fact that the human supplied rules will have inconsistencies and most likely will be incomplete.

Production management practice could greatly benefit from the experience of using fuzzy techniques in control engineering. Modelling of managerial decision processes in terms of rule-based and knowledge-based schemata may open new opportunities for effective production management.

Cost/benefit analysis

Cost/benefit analysis as a tool of production research in which all estimates are expressed in crisp monetary (time) values may lead to serious interpretation problems. As indicated by Ragade (1975), the benefits and costs do not have a statistical uncertainty so much as a meaning of incertitude. Furthermore, different estimates are likely to be evaluated by individuals or groups using imprecise data. Therefore, such cost/benefit estimates may have different degrees of reliability (confidence) depending upon the evaluators and the level of management structure at which they are developed. In addition, the cost of performing a large-scale cost/benefit analysis itself may be substantial and not justified.

More reliable (and simple) methods are needed to overcome the above difficulties. Neitzel and Hoffman (1980) proposed such methods based on fuzzy methodologies. The fuzzy cost/benefit analysis allows for the initial screening of the proposed alternatives in a short time and for the improved reliability of estimates. The method assumes that managers think and communicate in terms of linguistic descriptors (qualitative values) to a much better degree than in terms of numbers. Therefore, the estimates of direct materials, direct labor and overhead etc. are based on compatible methods of evaluation and given in the form of fuzzy descriptors (labels of corresponding fuzzy sets).

The process of identifying all items for which costs and benefits are relevant consists of:

(*a*) identification of the organizational structure of the system, its elements and their structural relationships

(*b*) development of the hierarchy of the affected activities or functions of these elements

(*c*) development of all the items within each of these activities for which costs/benefits should be estimated (e.g. personnel, materials, equipment, methods etc.).

The above process results in two hierarchical charts, one for the costs and one for the benefits. Each cost/benefit fuzzy estimate is modified by the corresponding fuzzy reliability estimate. For example, for the item material the cost/benefit values may be 'lower than medium'/'medium' respectively. Two values of reliability estimators are used: 'barely' and 'reasonable', with the understanding that 'barely' has greater fuzzification (uncertainty) effect than the 'reasonable' modifier.

The fuzzy algorithm, with operations necessary to combine the estimates, leads to a simple and reliable estimation of the total cost/benefit in the form of a fuzzy description, like: 'low'/'high', respectively. The described procedure improves the degree of confidence in the analysis since it takes into account the subjective estimations of the decision maker and incorporates the proportional contribution of each estimate at all levels of the organizational structure. By using fuzzy descriptors the estimation of the cost/benefit was simplified and the overall reliability of the analysis was improved.

Fuzzy mathematical programming

Mathematical programming has been applied to facilities location and layout design, production scheduling, and inventory control among other areas of production management. The utility of mathematical programming in these areas has been limited, however, for the reasons mentioned in the Introduction, including vagueness on the part of the decision maker with respect to the problem's objectives and constraints.

Fuzzy mathematical programming represents a way of incorporating vagueness into mathematical programs. For several reasons, including the potential usefulness of fuzzy mathematical programs and their wide variety (e.g. single objective or multi-objective, linear or non-linear etc.), much research has been accomplished in this area over the last ten years.

As noted by Bellman and Zadeh (1970), in a fuzzy environment goals and constraints can be viewed as being of the same nature. This view of a mathematical program is analogous to the view taken by many of the interactive algorithms for solving non-fuzzy multi-objective mathematical programs in which various objectives are constrained by aspiration levels (Evans 1984).

Tanaka *et al.* (1974) and Zimmermann (1976) were among the first researchers to address fuzzy linear programming. In Zimmermann's approach, the fuzziness inherent in the decision maker's preferences concerning the values achieved by the objective function and the constraint functions is modelled through the use of membership functions—one each for the objective function and each of the constraint functions. The desirability of a particular solution is then a fuzzy concept which can itself be modelled through the use of a membership function. This membership function represents the intersection of the fuzzy goal and fuzzy constraints. A solution which maximizes this membership function for desirability is called an 'optimal solution' to the fuzzy linear program. Zimmerman (1976) shows that the problem of maximization of the membership function is equivalent to another non-fuzzy linear program.

Another way in which fuzziness can be incorporated into a mathematical program is through the use of fuzzy numbers to represent constraint function coefficients, right-hand-sides, and objective function coefficients (Negoita and

Sularia 1976). In effect, this represents a more micro-oriented approach than the one described by Zimmermann (1976) since fuzziness is included at a 'lower-level' of the model. In addition, fuzziness has been incorporated into multi-objective mathematical programs (Zimmermann 1978, Baptistella and Ollero 1980, and Hannan 1981), and dynamic programs (Bellman and Zadeh 1970, Esogbue and Bellman 1983). Recently, Tanaka and Asai (1984) discussed fuzzy solutions to fuzzy linear programming problems.

Conclusions and needs for future research

Given the large body of work published during the last 20 years in the area of fuzzy sets and systems it is perhaps surprising that there have not been more examples of applications (actual or pseudo) of fuzzy methodologies in production management. There have been many publications which have suggested the incorporation of fuzziness into the quantitative techniques of OR/MS and many of these suggested approaches could prove useful for production research. Examples include the works of Watson *et al.* (1979) and Freeling (1980) in fuzzy decision analysis and Baas and Kwakernaak (1977) in multi-attribute utility theory. (See Zimmermann 1983 for a review of the uses of fuzzy sets in operations research).

As indicated by Negoita (1981), the importance of fuzzy concepts is that the decision makers are no longer required to state exactly the parameters of a model. Instead, they can introduce subjective evaluations of a non-distributional nature (as opposed to subjective probabilities) into the model. The work done by the Research Group in Integrated Automation (GRAI) in France (Pun 1977) on the practical applications of fuzzy formalism in production management systems proved that fuzzy methodologies are needed and can be effectively applied in that area of study. The use of fuzzy sets may extend the traditional approaches so that they can cope with, rather than delete, the ever present vagueness.

There are, however, many critical issues that must be further explored before the production management community accepts the use of fuzzy methodologies in their management practice. One of the most important issues involves how membership functions should be constructed. Some methods have already been proposed (e.g. exemplification, implicit analytical definition, use of statistics, or the relative preference method). Further research is needed to find more general methods of assessment which would be compatible with how people perceive and manipulate different imprecise categories and associated operators. In addition, research to investigate the sensitivity of the output of a fuzzy model as a function of the input membership functions could prove to be extremely useful.

Another important area of future research is the practical interpretation of the fuzzy output of fuzzy models. Defuzzification of the optimal decision may be as important as fuzzification of the relevant inputs. Therefore, the interpretation of fuzzy 'advice' output by a fuzzy model seems to be a prerequisite to wide acceptance of fuzzy concepts in the production management area.

L'emploi de méthodologies floues est une méthode efficace pour rendre compte de l'imprécision du jugement humain. Cette communication illustre les applications potentielles de méthodologies floues à divers stades de la gestion de la production incluant le développement d'un nouveau produit, le planning des installations, la gestion de la production humaine, la programmation de production et le contrôle d'inventaire, le contrôle de la qualité des procédés industriels, et l'analyse coût/bénéfice. De plus sont discutés de nouveaux terrains pour la recherche.

Der Einsatz verschwommenen Methodologien (engl. 'fuzzy methodologies') erlaubt es, der Vagheit des menschlichen Urteils angemessen Rechnung zu tragen. Diese Abhandlung beschreibt mögliche Anwendungen der verschwommenen Methodologien in verschiedenen Bereichen der Produktionsleitung einschließlich der Entwicklung neuer Produkte, Nutzungsplanung, Organisation der menschlichen Arbeitskraft, Arbeitsplanungs- und Lagerbestandsüberwachung, Qualitätskontrolle, Überwachung der Industrieverfahren und Kosten-/Nutzenanalyse. Zusätzlich werden wichtige Bereichte für weitere Forschungsvorhaben diskutiert.

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